

# Projectile Motion

*A two page physics handout, written by EulerDesk as an accessibility sample.*

## The setup

A ball is launched from level ground with speed  $v_0$  at an angle  $\theta$  above the horizontal. If we ignore air resistance, the only force on the ball after launch is gravity, which pulls it down with acceleration  $g$ . The horizontal and vertical motions can then be treated separately.

## Position at time $t$

The horizontal velocity never changes, so the horizontal position grows steadily with time:

$$x(t) = v_0 \cos \theta t$$

The vertical velocity starts at  $v_0 \sin \theta$  and falls at the rate  $g$ , which gives:

$$y(t) = v_0 \sin \theta t - \frac{1}{2}gt^2$$

## Three useful results

Setting  $y(t) = 0$  gives the time of flight  $T$ . Putting  $T$  into  $x(t)$  gives the range  $R$ , and the top of the path gives the maximum height  $H$ .

|                       |                                  |
|-----------------------|----------------------------------|
| <b>Time of flight</b> | $T = \frac{2v_0 \sin \theta}{g}$ |
|-----------------------|----------------------------------|

|              |                                    |
|--------------|------------------------------------|
| <b>Range</b> | $R = \frac{v_0^2 \sin 2\theta}{g}$ |
|--------------|------------------------------------|

|                       |                                      |
|-----------------------|--------------------------------------|
| <b>Maximum height</b> | $H = \frac{v_0^2 \sin^2 \theta}{2g}$ |
|-----------------------|--------------------------------------|

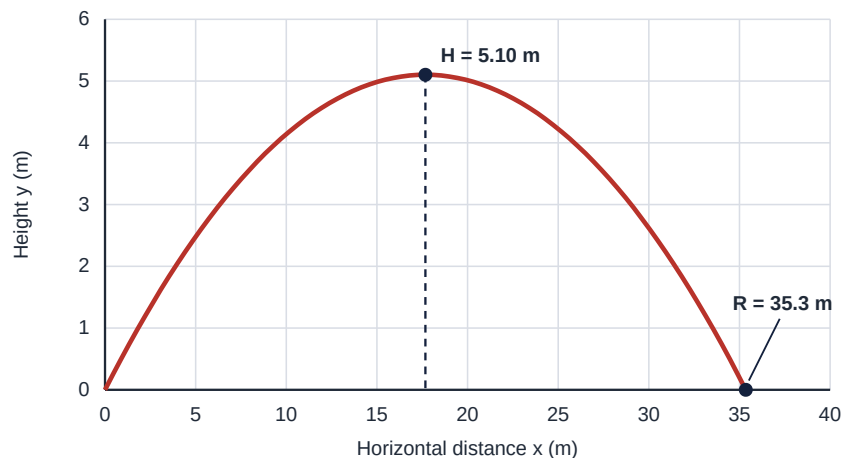


*Figure 1. Path of a ball launched at 20 m/s and 30°. It reaches a maximum height of 5.10 m and lands 35.3 m away.*

## Worked example

A ball is launched at  $v_0 = 20 \text{ m/s}$  and  $\theta = 30^\circ$ . Take  $g = 9.8 \text{ m/s}^2$ . Find the time of flight, the range and the maximum height.

1. Time of flight

$$T = \frac{2 \times 20 \times \sin 30^\circ}{9.8} = 2.04 \text{ s}$$

2. Range

$$R = \frac{20^2 \times \sin 60^\circ}{9.8} = 35.3 \text{ m}$$

3. Maximum height

$$H = \frac{20^2 \times \sin^2 30^\circ}{2 \times 9.8} = 5.10 \text{ m}$$

## Changing the launch angle

Table 1 keeps  $v_0 = 20 \text{ m/s}$  and changes only the angle.

Table 1. Flight time, maximum height and range for  $v_0 = 20 \text{ m/s}$  and  $g = 9.8 \text{ m/s}^2$

| Launch angle | Time of flight (s) | Maximum height (m) | Range (m) |
|--------------|--------------------|--------------------|-----------|
| 15°          | 1.06               | 1.37               | 20.4      |
| 30°          | 2.04               | 5.10               | 35.3      |
| 45°          | 2.89               | 10.2               | 40.8      |
| 60°          | 3.53               | 15.3               | 35.3      |
| 75°          | 3.94               | 19.0               | 20.4      |

Two things stand out. The range is largest at  $45^\circ$ , and angles that add up to  $90^\circ$ , such as  $30^\circ$  and  $60^\circ$ , give the same range. The higher angle keeps the ball in the air longer and sends it higher.

## Try it yourself

1. At what launch angles would the ball land 20.4 m away?
2. If  $v_0$  doubled, how would the range change?

Answers:  $15^\circ$  or  $75^\circ$ . The range would become four times larger.