Projectile Motion

A two page physics handout, written by EulerDesk as an accessibility sample.

The setup

A ball is launched from level ground with speed v_0 at an angle θ above the horizontal. If we ignore air resistance, the only force on the ball after launch is gravity, which pulls it down with acceleration g. The horizontal and vertical motions can then be treated separately.

Position at time t

The horizontal velocity never changes, so the horizontal position grows steadily with time:

$$x(t) = v_0 \cos\theta \, t$$

The vertical velocity starts at $v_0 \sin\theta$ and falls at the rate g, which gives:

$$y(t) = v_0 \sin\theta \, t - \tfrac{1}{2} g t^2$$

Three useful results

Setting y(t) = 0 gives the time of flight T. Putting T into x(t) gives the range R, and the top of the path gives the maximum height H.

Time of flight $$T = \frac{2 v_0 \sin\theta}{g}$$

Range $$R = \frac{v_0^2 \sin 2\theta}{g}$$

Maximum height $$H = \frac{v_0^2 \sin^2\theta}{2g}$$

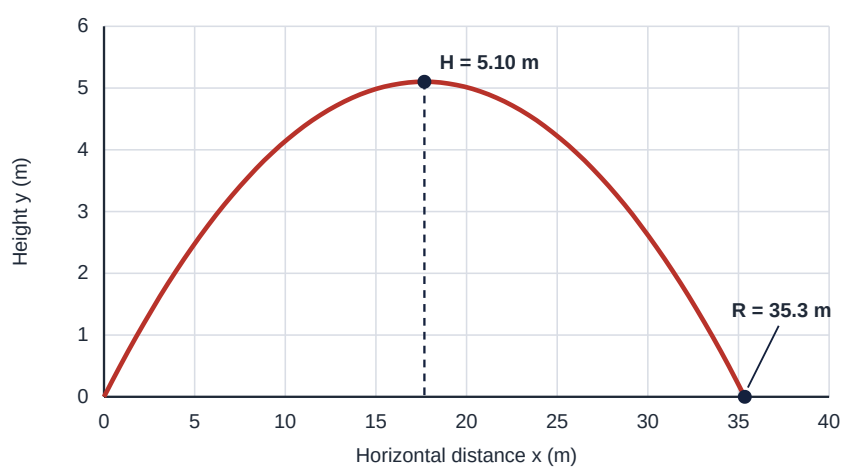

Figure 1. Path of a ball launched at 20 m/s and 30°. It reaches a maximum height of 5.10 m and lands 35.3 m away.

Worked example

A ball is launched at v_0 = 20 m/s and θ = 30°. Take g = 9.8 m/s². Find the time of flight, the range and the maximum height.

1. Time of flight
$$T = \frac{2 \times 20 \times \sin 30^\circ}{9.8} = 2.04 \text{ s}$$
2. Range
$$R = \frac{20^2 \times \sin 60^\circ}{9.8} = 35.3 \text{ m}$$
3. Maximum height
$$H = \frac{20^2 \times \sin^2 30^\circ}{2 \times 9.8} = 5.10 \text{ m}$$

Changing the launch angle

Table 1 keeps v_0 = 20 m/s and changes only the angle.

Table 1. Flight time, maximum height and range for v_0 = 20 m/s and g = 9.8 m/s²

Launch angle	Time of flight (s)	Maximum height (m)	Range (m)
15°	1.06	1.37	20.4
30°	2.04	5.10	35.3
45°	2.89	10.2	40.8
60°	3.53	15.3	35.3
75°	3.94	19.0	20.4

Two things stand out. The range is largest at 45°, and angles that add up to 90°, such as 30° and 60°, give the same range. The higher angle keeps the ball in the air longer and sends it higher.

Try it yourself

1. At what launch angles would the ball land 20.4 m away?
2. If v_0 doubled, how would the range change?

Answers: 15° or 75°. The range would become four times larger.

Sample document by EulerDesk, eulerdesk.com